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Entropy-Based weighting applied to normal boundary intersection approach: the vertical turning of martensitic gray cast iron piston rings case

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ABSTRACT. In practical situations, solving a given problem usually calls for the systematic and simultaneous analysis of more than one objective function. Hence, a worthwhile research question may be posed thus: In multiobjective optimization, what can facilitate the decision maker in choosing the best weighting? Thus, this study attempts to propose a method that can identify the optimal weights involved in a multiobjective formulation. Our method uses functions of Entropy and Global Percentage Error as selection criteria of optimal weights. To demonstrate its applicability, we employed this method to optimize the machining process for vertical turning martensitic gray cast iron piston rings, maximizing the productivity and the life of cutting tool and minimizing the cost, using feed rate and rotation of the cutting tool as the decision variables. The proposed optimization goals were achieved with feed rate = 0.35 mm rev^{-1} and rotation = 248 rpm. Thus, the main contributions of this study are the proposal of a structured method, differentiated in relation to the techniques found in the literature, of identifying optimal weights for multiobjective problems and the possibility of viewing the optimal result on the Pareto frontier of the problem. This viewing possibility is very relevant information for managing processes more efficiently.

Keywords: pareto frontier, optimal weights, mixture design of experiments.

Ponderação baseada em Entropia aplicada à abordagem de interseção normal à fronteira: o caso do torneamento vertical de anéis de pistão de ferro fundido martensítico

RESUMO. Em situações práticas, normalmente se tem mais de uma função objetivo a ser analisada de maneira sistemática e simultânea para a resolução de determinado problema. Desta forma, surge a seguinte questão de pesquisa: como auxiliar o tomador de decisão na escolha da melhor ponderação ao se trabalhar com otimização multiobjetivo? O presente trabalho propõe um método que possa identificar os pesos ótimos envolvidos em uma formulação multiobjetivo utilizando as funções de Entropia e de Erro Percentual Global (EPG) como critérios de seleção. Empregou-se o método para otimizar o processo de torneamento vertical de anéis de pistão de ferro fundido cinzento martensítico, maximizando a produtividade e a vida da ferramenta de corte e minimizando o custo, usando como variáveis de decisão o avanço e a rotação da peça. Os objetivos de otimização propostos foram alcançados com avanço = 0,35 mm $rot.^{-1}$ e rotação = 248 rpm. Assim, as principais contribuições do presente trabalho foram a proposição de um método estruturado, diferenciado em relação às técnicas encontradas na literatura, para a identificação de pesos ótimos em problemas multiobjetivos e a possibilidade de visualização do resultado ótimo na fronteira de Pareto do problema, sendo esta última uma informação de grande relevância para uma gestão mais eficiente dos processos.

Palavras-chave: fronteira de pareto, pesos ótimos, arranjo de misturas.

Introduction

Optimization techniques, in recent years, have evolved greatly, finding wide application in various types of industries, mainly because making decision about complex problems involves process optimization and engineering design (HEJAZI et al., 2014). They are now capable of solving ever larger and more complex problems, thanks to a new generation of powerful computers.

According to Rao (2009), optimization is the act, in any given circumstance, of obtaining the best result. In this context, the main purpose of decision making in industrial processes is to minimize the effort required to develop a specific task or to maximize the desired benefit. The effort required or the benefit desired in any practical situation can be expressed as a function of certain decision variables. This function is known as the objective function.

In practical situations, however, solving a given problem usually calls for the systematic and simultaneous analysis of more than one objective function, resulting in multiobjective optimization (HUANG et al., 2006, ADEYEMO; OLOFINTOYE, 2014).

In multiobjective problems, it is very unusual that all functions are minimized simultaneously by one optimal solution $x^{\star}$. Indeed, the multiple objectives have conflicts of interest (RAO, 2009). What becomes of great relevance to these types of problems, according to Rao (2009), is the concept of a Pareto-optimal solution, also called a compromise solution. The author refers to a feasible solution $x^{\star}$ as Pareto-optimal if no other feasible solution z exists such that $f_i(z) \leq f_i(\mathrm{x}^*)$, $i = 1,2,..., m$, with $f_j(z) < f_i(\mathrm{x}^*)$ in at least one objective j. Pareto-optimal solutions occur because of the conflicting nature of the objectives, where the value of any objective function cannot be improved without impairing at least one of the others. In this context, a trade-off represents giving up one objective to improve another (ESKELINEN; MIETTINEN, 2011).

The purpose of multiobjective optimization methods is to offer support and ways to find the best compromise solution, in which the decision maker and his preference information play important roles (ESKELINEN; MIETTINEN, 2011). A decision maker, according to Eskelinen and Miettinen (2011), is an expert in the domain of the problem under consideration and who typically is responsible for the final solution. In order to define the relative importance of each objective function, the decision maker must assign them different weights.

Because a characteristic property of multiobjective optimization is the problem of weighting objective functions, how a decision maker is involved with the solution of this problem is the basis for its classification. According to Hwang and Masud (1979) and Miettinen (1999), the classes are: 1. no-preference methods: methods where no articulation of preference information is made; 2. a priori methods: methods where a priori articulation of preference information is used, i.e., the decision-maker selects the weighting before running the optimization algorithm; 3. interactive methods: methods where progressive articulation of preference information is used, i.e., the decision maker interacts with the optimization program during the optimization process; and 4. a posteriori methods: methods where a posteriori articulation of preference information is used, i.e., no weighting is specified by the user before or during the optimization process. However, as no classification can be complete, these classifications are not absolute. Overlapping and combinations of classes are possible and some methods can be considered to belong to more than one class (HWANG; MASUD, 1979). This paper considers the a posteriori method in consonance with generate first-choose later approach (MESSAC; MATTSON, 2002).

A multiobjective problem is generally solved by reducing it to a scalar optimization problem; hence, the term scalarization. Scalarization is the converting of the problem, by aggregation of the components of the objective functions, into a single or a family of single objective optimization problems with a real-valued objective function (HWANG; MASUD, 1979). The literature reports different scalarization methods. The most common is the weighted sum method.

The weighted sum method is widely employed to generate the trade-off solutions for nonlinear multiobjective optimization problems. According to Shin et al. (2011), a bi-objective problem is convex if the feasible set X is convex, as are the functions. When at least one objective function is not convex, the bi-objective problem becomes non-convex, generating a non-convex and even unconnected Pareto frontier. The main consequence of a non-convex Pareto frontier is that points on the concave parts of the trade-off surface will not be estimated (DAS; DENNIS, 1997). This instability happens because the weighted sum is not a Lipshitzian function of the weight w (VAHIDINASAB; JADID, 2010). Another drawback to the weighted sums is related to the uniform spread of Pareto-optimal solutions. Even if a uniform spread of weight vectors are used, the Pareto frontier will be neither equispaced nor evenly distributed (DAS; DENNIS, 1997, VAHIDINASAB; JADID, 2010).

Given its drawbacks, the weighted sum method is not used in this paper. Instead, the normal boundary intersection method is employed (NBI), as proposed by Das and Dennis (1998). These authors proposed the NBI method to overcome the disadvantages of the weighted sum method, showing that the Pareto surface is evenly distributed independent of the relative scales of the objective functions. It is due to this feature that this study uses the NBI method to build the Pareto frontier.

In the multiobjective optimization process, the decision maker plays an important role, for it is the decision maker that eventually obtains a single solution to be used in his original multidisciplinary decision-making problem. Hence, a worthwhile research question may be posed thus: In multiobjective optimization, what can facilitate the decision maker in choosing the best weighting?

In answering such a question, we propose the use two objectively defined selection criteria: Shannon's Entropy Index (SHANNON, 1948) and Global Percentage Error (GPE). Entropy can be defined as a measure of probabilistic uncertainty. Its use is indicated in situations where the probability distributions are unknown, in search of diversification. Among the many other desirable properties of Shannon's Entropy Index, we highlight the following: 1) Shannon's measure is nonnegative, and 2) its measure is concave. Property 1 is desirable because the Entropy Index ensures non-null solutions. Property 2 is desirable because it is much easier to maximize a concave function than a non-concave one (FANG et al., 1997). The GPE, as its name declares, is an error index. In this case, we want to evaluate the distance of the determined Pareto optimal solution from its ideal value.

Thus, this study attempts to propose a method that can identify the optimal weights involved in a multiobjective formulation. Our method uses both a Normal Boundary Intersection (NBI) approach along with Mixture Design of Experiments and, as selection criteria of optimal weights, uses the functions of Entropy and Global Percentage Error (GPE).

To demonstrate its applicability, we employed this method to optimize the machining process for vertical turning martensitic gray cast iron piston rings. This is a relatively complex machining process. A roughing operation, it is conducted simultaneously on the outer and inner diameters of the parts. The machining is carried out using two twin cutting tools with special geometry; abundant cooling is provided throughout the cutting process (SEVERINO et al., 2012), which is important to increase machining performance (ÇOLAK, 2014, RODRIGUES et al., 2014). According to Wang et al. (2007), of the many types of cast iron used by the auto industry, gray cast iron is the most used. This is the main reason to study this process. What the industry needs is a conditioning process that improves the machinability of martensitic gray cast iron in vertical turning operations of piston rings. To meet this need, this paper proposes an optimization of cutting conditions, maximizing the productivity and the life of cutting tool and minimizing the cost, using feed rate and rotation of the cutting tool as the decision variables, according to proposed by Severino et al.(2012). It is important to note that in the vertical turning process we are unable to change directly the cutting speed being this parameter considered stable when working within the manufacturer's specifications. Because of this process feature, cutting speed is not used as a variable in the present study.

Theoretical background

Design of experiments

According to Montgomery (2001), an experiment can be defined as a test or a series of tests in which purposeful changes are made to the input variables of a process, aiming thereby to observe how such changes affect the responses. Design of Experiments (DOE) is then defined as the process of planning experiments so that appropriate data is collected and then analyzed by statistical methods, leading to valid and objective conclusions (MONTGOMERY, 2001).

According to Montgomery (2001), the three basic principles of DOE are randomization, replication, and blocking. Randomization is the implementation of experiments in a random order such that the unknown effects of the phenomena are distributed among the factors, thereby increasing the validity of the research. Replication is the repetition of the same test several times, creating a variation in the response that is used to evaluate experimental error. The blocking should be used when it is not possible to maintain the homogeneity of the experimental conditions. This technique allows us to evaluate whether the lack of homogeneity affects the results.

The steps of DOE are (MONTGOMERY, 2001): recognition and problem statement; choice of factors, levels and variations; selection of the response variable; choice of experimental design; execution of the experiment; statistical analysis of data; conclusions and recommendations.

Regarding the experimental projects, the most widely used techniques include the full factorial design, the fractional factorial design, the arrangements of Taguchi, response surface methodology and experiments of mixtures Montgomery (2001).

Response surface methodology

Response surface methodology (RSM) is a collection of mathematical and statistical tools used to model and analyze problems in which responses of interest are influenced by several variables. The objective of RSM is to optimize these responses (MONTGOMERY, 2001).

For most industrial processes, the relationships between responses and independent variables are unknown, so RSM seeks to find a suitable approximation to represent the responses of interest as a function of these variables. To describe such relationships, researchers generally use polynomial functions. Thus, if a response is well modeled by a linear function, the approximate ratio can be represented by the following first order model (MONTGOMERY, 2001):

$$y(x) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \ldots + \beta_k x_k + \varepsilon \qquad (1)$$

where:
$y(x)$ – Response of interest;
x_i – Independent variables;
β_i – Coefficients to be estimated;
k – Number of independent variables;
ε – Experimental error.

If the answer presents curvature, then a polynomial of a higher degree must be used as the second-order model:

$$y(x) = \beta_0 + \sum_{i=1}^{k} \beta_i x_i + \sum_{i=1}^{k} \beta_{ii} x_i^2 + \sum_{i<j}\sum \beta_{ij} x_i x_j + \varepsilon \qquad (2)$$

Almost all problems of response surface use either one or both models presented. In addition, while it is unlikely for a polynomial model to behave as a proper approach for the entire experimental space covered by the independent variables, such models have been shown to be effective for a specific region (MONTGOMERY, 2001).

As one objective of RSM is to optimize the response, it is recommended, whenever possible, to represent it by second-order models, since the curvature shown by these defines the location of a stationary point. Therefore, when the response of interest presents a linear behavior, the information from the first-order model should be used to find the region of curvature. This should be done using the vector gradient method (steepest descent/ascent) (MONTGOMERY, 2001).

The estimation of coefficients, defined by the model of Equations (1) and (2), is typically performed using the Ordinary Least Squares method. With this method, an approximate function that connects the response of interest with the process variables would be constructed (MONTGOMERY, 2001). After constructing the model, the statistical significance of the same should be verified through Analysis of Variance (ANOVA). ANOVA, apart from revealing the significance of the model as a whole, permits one to check which of the model's terms are significant and which may be neglected. The fit is represented by the coefficient of determination (R^2), which represents the percentage of the observed data in the response that can be explained by the mathematical model. Associated with this coefficient is the adjusted R^2, which is an alternative to measuring the coefficient of determination. The adjusted R^2 penalizes the inclusion of less explanatory regressors, avoiding the tendency to overestimate the current variation in the data, taken by R^2, when a larger number of variables are inserted. For the modeling of the response surface functions, the experimental arrangement most often used for data collection is the central composite design (CCD) (MONTGOMERY, 2001). CCD, for *k* factors, is a matrix formed by three distinct groups of experimental elements: a full factorial 2^k or fractional 2^{k-p}, where *p* is the desired fraction of the experiment; a set of central points (cp); and, in addition, a group of extreme levels called axial points, given by *2k*. The number of experiments required is given by the sum: $2^{k \text{ or } (k-p)} + cp + 2k$. In CCD, the axial points are within a distance α of the central points, being $\alpha = (2^k)^{1/4}$ (BOX; DRAPER, 1987).

Mixture design of experiments

In mixture design of experiments, the factors are the ingredients or components of a mixture, and consequently, their levels are not independent. For example, if $x_1, x_2, \ldots, x_p$ indicate the proportions of *p* components of a mixture, then (MONTGOMERY, 2001):

The constraint of Equation (3) can be viewed graphically in Figure 1 for *k=2* and *k=3* components. With two components, the experimental region for the mixture experiments considers all values along the line $x_1 + x_2 = 1$ (Figure 1a). In the case of three components, this region is the area bounded by the triangle seen in Figure 1b, where the vertices correspond to the neat blends, the sides to the binary mixtures, and the triangular region to the complete mixtures (MONTGOMERY, 2001). The existence of these features makes it quite necessary that mixture experiments be planned and conducted through specific arrangements and that those most used be, in this context, the simplex arrangements (CORNELL, 2002).

$$0 \le x_i \le 1 \qquad i = 1, 2, \ldots, p \text{ and } x_1 + x_2 + \ldots + x_p = 1 \tag{3}$$

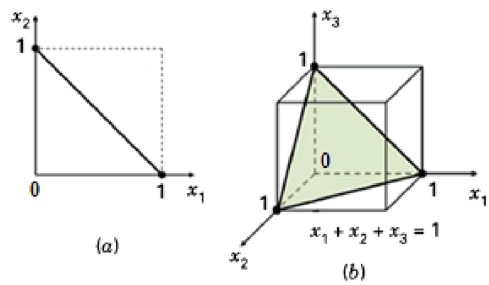


Figure 1. Experimental region for mixture experiments (MONTGOMERY, 2001).

The simplex arrangements are defined as a configuration in which the vertices of the triangle represent the maximum proportion of the input variables and the interior points of this triangle describe the possible combinations of these variables. A simplex arrangement can be of two main types: the simplex lattice arrangement and the simplex centroid arrangement. A graphical representation of simplex arrangement is shown in Figure 2.

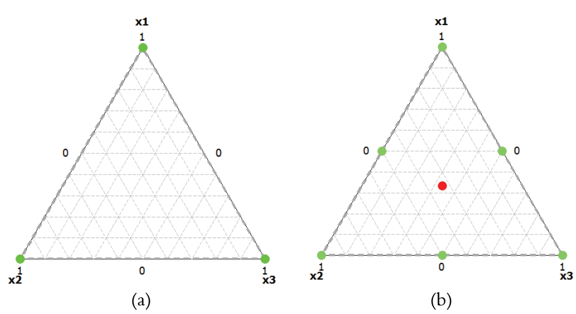


Figure 2. Mixture arrangement: (a) simplex lattice. (b) simplex centroid.

In a simplex lattice, the *k* input variables define points whose proportions are assumed to take into consideration *m + 1* equally spaced values between 0 and 1, such that (MONTGOMERY, 2001):

All possible combinations (mixtures) of the proportions of Equation (4) are used, where *m* is the lattice degree of the arrangement. The total number of experiments (*N*) is given by:

$$x_i = 0, \frac{1}{m}, \frac{2}{m}, \ldots, 1 \qquad i = 1, 2, \ldots, k \tag{4}$$

$$N = \frac{(k+m-1)!}{m!(k-1)!} \tag{5}$$

A disadvantage with the simplex arrangements concerns the fact that most experiments occur at the borders of the array. This results in few points of the internal part being tested. Thus, it is recommended, whenever possible, to increase the number of experiments by adding internal points to the arrangements, as the central points and the axial points. In the case of arrangements of mixtures, it is noteworthy that the central points correspond to the centroid itself.

Regarding the mathematical models used for the representation of the responses, it appears that the mixture models have some differences from polynomials employed in RSM, mainly due to the existence of the constraint $\sum_{i=1}^{n} w_i = 1$. The models of mixtures' more widespread standard forms are (MONTGOMERY, 2001):

Linear:

$$E(x) = \sum_{i=1}^{q} \beta_i x_i \tag{6}$$

Quadratic:

$$E(x) = \sum_{i=1}^{q} \beta_i x_i + \sum \sum_{i<j}^{q} \beta_{ij} x_i x_j \tag{7}$$

Full cubic:

$$E(x) = \sum_{i=1}^{q} \beta_i x_i + \sum \sum_{i<j}^{q} \beta_{ij} x_i x_j + \sum \sum_{i<j}^{q} \delta_{ij} x_i x_j (x_i - x_j) + \sum \sum_{i<j<k} \sum^{q} \beta_{ijk} x_i x_j x_k \tag{8}$$

Special cubic:

$$E(x) = \sum_{i=1}^{q} \beta_i x_i + \sum \sum_{i<j}^{q} \beta_{ij} x_i x_j + \sum \sum_{i<j<k} \sum^{q} \beta_{ijk} x_i x_j x_k \tag{9}$$

The distinctive shapes of the previous functions makes Equations (6) - (9) to be called canonical polynomials of mixtures or Scheffé's polynomials (CORNELL, 2002). The estimation of the coefficients is done in a similar way to that used in RSM, the same occurring for the analysis of variance, analysis of residuals and other statistical tests.

Normal boundary intersection approach

The normal boundary intersection method (NBI) is an optimization routine developed to find Pareto-optimal solutions evenly distributed for a non-linear multiobjective problem (DAS; DENNIS, 1998). The first step in the NBI method comprises the establishment of the payoff matrix Φ, based on the calculation of the individual minimum of each objective function. The solution that minimizes the *i*-th objective function $f_i(x)$ can be represented as $f_i^*(x_i^*)$. When it replaces the optimal individual x_i^* in the remaining objective functions, we have $f_i(x_i^*)$. In matrix notation, the payoff matrix Φ, for *m* objective functions, can be written as (VAHIDINASAB; JADID, 2010, BRITO et al., 2014):

$$\Phi = \begin{bmatrix} f_1^*(x_1^*) & \cdots & f_1(x_i^*) & \cdots & f_1(x_m^*) \\ \vdots & \ddots & & & \vdots \\ f_i(x_1^*) & \cdots & f_i^*(x_i^*) & \cdots & f_i(x_m^*) \\ \vdots & & & \ddots & \vdots \\ f_m(x_1^*) & \cdots & f_m(x_i^*) & \cdots & f_m^*(x_m^*) \end{bmatrix} \tag{10}$$

The values of each row of the payoff matrix Φ, which consist of minimum and maximum values of the *i*-th objective function $f_i(x)$, can be used to normalize the objective functions, generating the normalized matrix Φ, such as (BRITO et al., 2014):

$$\bar{f}(x) = \frac{f_i(x) - f_i^U}{f_i^N - f_i^U} \quad i = 1, \ldots, m \tag{11}$$

This procedure is mainly used when the objective functions are written in terms of different scales or units. The Utopia point is obtained by writing a vector with all the individual minimum, $f^U = [f_1^*(x_1^*) \ \ldots, \ f_i^*(x_i^*) \ \ldots, \ f_m^*(x_m^*)]^T$. Joining the maximum values of each objective function, $f^N = [f_1^N \ \ldots, \ f_i^N \ \ldots, \ f_m^N]^T$, we have a set called Nadir point.

According to Vahidinasab and Jadid (2010), the convex combinations of each row of the payoff matrix form the Convex Hull of Individual Minimum (CHIM). A uniform displacement of any point along the Utopia line does not lead to a good distribution of the Pareto points. The anchor point corresponds to the solution of single optimization problem $f_i^*(x_i^*)$. The *m* anchor points, depending on the number of objective functions, are connected by the Utopia line (UTYUZHNIKOV et al., 2009).

Considering now a convex weighting *w*, such as Φw_i represents a point in the CHIM. Let $\hat{n}$ denote the unit normal direction (a column vector of ones) to the CHIM at the point Φw_i towards the origin; then, $\Phi w + D\hat{n}$, with $D \in \mathbb{R}$, represents the set of points on that normal (SHUKLA; DEB, 2007, JIA; IERAPETRITOU, 2007). The Figure 3 graphically represents the NBI method, wherein: a, b and e are calculated as Φw_i.

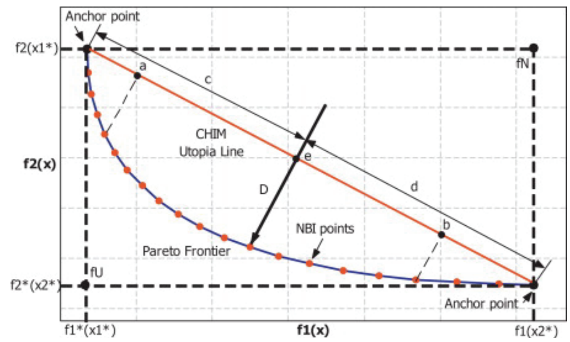


Figure 3. Graphical description of NBI method (BRITO et al., 2014).

The intersection point between the normal and the nearest boundary of the feasible region from origin corresponds to the maximization of distance between the Utopia line and the Pareto frontier. This optimization problem can be solved iteratively for different values of *w*, creating a Pareto frontier uniformly distributed. A common choice for *w* was suggested by Das and Dennis (1998) and Jia and Ierapetritou (2007) as $w_n = 1 - \sum_{i=1}^{n-1} w_i$. Then, the optimization problem can be written as (SHUKLA; DEB, 2007):

$$\begin{aligned} &\underset{(\mathbf{x}, D)}{\text{Max}} \ D \\ &s.t.: \ \bar{\Phi} w + D\hat{n} = \bar{F}(\mathbf{x}) \\ &\qquad \mathbf{x} \in \Omega \end{aligned} \tag{12}$$

Process of vertical turning dual piston rings

The main features of martensitic gray cast iron are its low melting point (PRADHAN et al., 2007), good fluidity, and high resistance to wear (HEJAZI et al., 2009). However, the machinability of martensitic gray cast iron is compromised by its chemical composition, which include graphitizing elements. It also includes others elements that impair its machinability: carbide-forming elements and hard abrasives, such as niobium, tungsten, vanadium, chromium, titanium and molybdenum (Table 1). Besides, martensitic microstructure, consisting of graphite shafts and a tempered martensitic matrix (Figure 4), is an impairment to machinability too. This material has an average hardness of 40 HRC (SEVERINO et al., 2012).

Table 1. Chemical Composition of Martensitic Cast Iron (SEVERINO et al., 2012).

%	C	Si	Mn	P	S	Cu	Cr	Ni	Mo	Ti	V	W	Nb
Mean	3.4	4.3	0.85	0.18	0.08	1.15	0.4	1.05	1.5	0.15	0.75	0.8	0.65

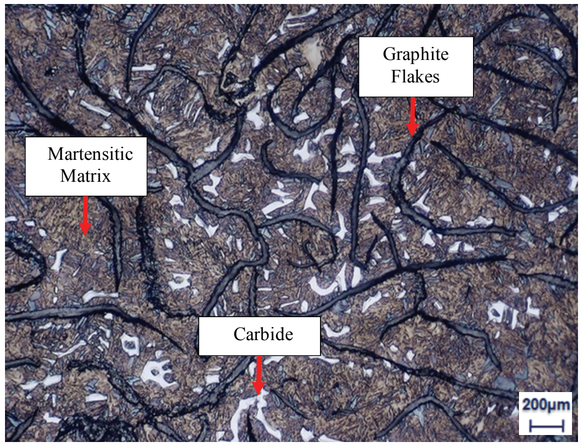
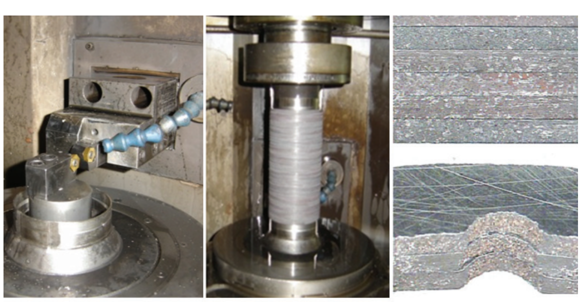


Figure 4. Tempered martensitic matrix (SEVERINO et al., 2012).

Because the aforementioned features, the cast iron piston rings manufactured in sand molds have an extremely rough and abrasive martensitic surface.

Deviations in shape and excessive surface roughness, both caused by the smelting process, must be corrected. To do so, rings with diameters of 81.60 mm and 1.95 mm thickness undergo a process of dual vertical turning (Figure 5-a) in packets of 77 units (Figure 5-b), kept under constant pressure and aligned with a recess in the internal diameter (Figure 5-c).The packets are turned vertically by the simultaneous action of two identical, cemented carbide tools.



(a) (b) (c)

Figure 5. Vertical turning of Martensitic cast iron piston rings (SEVERINO et al., 2012).

Metamodeling

Many of the techniques used in these strategies rely, at least in one of its stages, on imprecise and subjective elements. Hence, the analysis of weighting methods for multiple responses demonstrates that, since a large portion of strategies still use elements liable to error, significant contributions can still be made.

Our effort to contribute to this topic consists of developing an alternative for the identification of optimal weights in problems of multiobjective optimization. Statistical methods based on DOE are important techniques to model objective functions. Indeed, for most industrial processes, the mathematical relationships are unknown. The insertion of optimization algorithms takes place during the step of identifying optimal solutions for the responses and for the weights—after they have been modeled by the statistical techniques mentioned above. The GRG algorithm is used by the Excel® Solver function. The NBI approach is also used in the search for optimal weights, using as selection criteria the functions Entropy and Global Percentage Error (GPE).

To reach the weighting methodology proposed in this study, the following procedures are used:

Step 1: Experimental design:

Establishment of the experimental design and execution of experiments in random order.

Step 2: Modeling the objective functions:

Definition of equations using the experimental data.

Step 3: Formulation of the problem of multiobjective optimization:

The objective functions (productivity, life of cutting tool and cost) are aggregated into a formulation of multiobjective optimization, by NBI approach, like in Equation (12).

Step 4: Definition of mixtures arrangement:

In order to set the weights to be used in the optimization routine described in Step 3, a mixtures arrangement is done using Minitab® 16. Due to the constraint $\sum_{i=1}^{n} w_i = 1$, the use of the mixtures arrangement is feasible.

Step 5: Solution of the optimization problem:

The optimization problem of Step 3 is solved for each experimental condition defined in Step 4.

Step 6: Calculation of Global Percentage Error (GPE) and Entropy:

GPE of Pareto-optimal responses is calculated, defining how far the analyzed point is from the objective function's ideal value, namely the target. The GPE is calculated through expression (ROCHA et al., 2015):

$$GPE = \sum_{i=1}^{m} \left| \frac{y_i^*}{T_i} - 1 \right| \tag{13}$$

where:

y_i^* – Value of the Pareto-optimal responses;

T_i – Targets defined;

m – Number of objectives.

In order to diversify the weights of multiobjective optimization, Shannon's Entropy

Index is calculated using the Pareto-optimal responses, through the expression:

$$S(x) = -\sum_{i=1}^{m} w_i \ln(w_i) \quad (14)$$

Step 7: Modeling of GPE and Entropy:

The canonical polynomial mixtures for GPE and for Entropy is determined using as data the results of the calculations from Step 6.

Step 8: Defining the optimal weights:

To achieve the optimal weights, we consider the following routine:

$$\begin{aligned} & Max \quad \xi = \frac{Entropy}{GPE} \\ & s.t.: \quad \sum_{i=1}^{n} w_i = 1 \\ & \qquad 0 \le w_i \le 1 \end{aligned} \quad (15)$$

The Figure 6 shows the proposal step-by-step. With this routine, we maximize the relation between Entropy and GPE. These parameters are, in this proposal, the selection criteria for optimal weights.

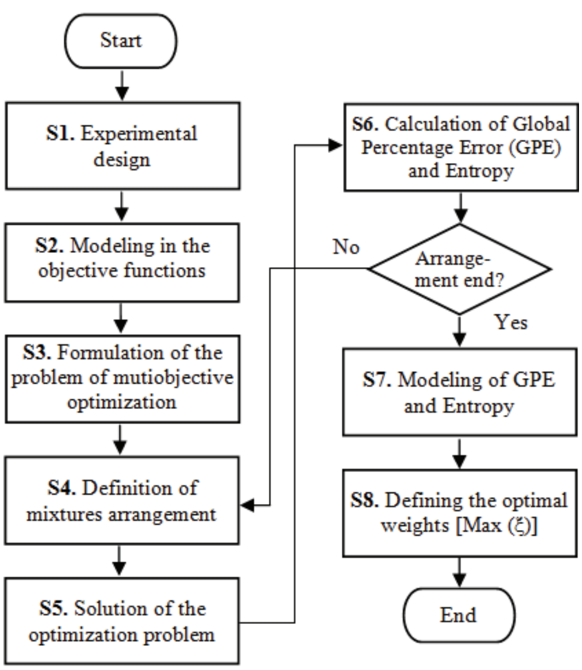


Figure 6. Optimal Weights Identification Process.

Once this procedure has been performed and the optimal weights have been achieved, the multiobjective optimization should be performed to reach the optimal values for decision variables in the original problem.

Implementation of the proposed method

In order to apply the method proposed in this study, we used the experimental data presented in Severino et al. (2012). The authors aimed to optimize, with the application of DOE, a process of vertical turning to determine the condition that led to a maximum life of the cutting tool (mm), high productivity (parts hour^{-1}), and minimum cost (US\$ part^{-1}). Using feed rate (mm rev^{-1}) and rotation (rpm) of the cutting tool as the decision variables, a full factorial design 2^2 (2 factors and 2 levels) was performed, with 4 axial points and 5 center points, as suggested by Box and Draper (1987), generating 13 experiments (Table 2). The criteria for the end of tool life was based on the maximum flank wear (= 0.3mm), the breakdown of the tool, and the chipping of the piston ring (SEVERINO et al., 2012). All operational parameters of the vertical turning machine followed the manufacturer's recommendation.

Table 2. CCD for life of the cutting tool, productivity and cost (SEVERINO et al., 2012).

N	Feed (mm rev^{-1})	Rotation (rpm)	Life of cutting tool (mm)	Productivity (part hour^{-1})	Cost (US\$ part^{-1})
1	0.32	235	2,102	1,523	0.04686
2	0.38	235	2,853	1,712	0.03682
3	0.32	275	1,802	1,677	0.04474
4	0.38	275	1,501	1,847	0.04413
5	0.31	255	1,652	1,555	0.05019
6	0.39	255	1,802	1,813	0.04117
7	0.35	227	2,853	1,588	0.04047
8	0.35	283	1,952	1,807	0.03985
9	0.35	255	3,153	1,714	0.03562
10	0.35	255	3,003	1,713	0.03620
11	0.35	255	3,303	1,716	0.03509
12	0.35	255	2,703	1,709	0.03755
13	0.35	255	2,853	1,711	0.03684

Data from the vertical turning process of piston rings were used to obtain the total machining time, production rate, and the machining cost per part. These responses of interest were obtained in the various cutting conditions suggested by the CCD (Table 2), using life of the cutting tool as a characteristic of output that was observed during the tests. The productivity (P_R) and cost (K_P) were obtained, as suggested in Severino et al. (2012) and Paiva et al. (2007), from Equations (16), (17), and (18). According to these authors, the productivity and cost parameters are useful processes' efficiency measures.

$$P_R = 60 \times T_t^{-1} \quad (16)$$

$$T_t = C_t + \left(t_a + t_s \frac{t_p}{Z}\right) + \left(\frac{C_t}{T} - \frac{1}{Z}\cdot\right) t_{ft} \quad (17)$$

$$K_p = \left(\frac{T_t}{60} - \frac{1}{Z}\right).\left(S_h + S_m\right) + \frac{C_t}{60}\left(S_h + S_m\right) + \frac{C_t}{T}\left[\left(\frac{V_{si}}{N_{fp}} + \frac{K_{pi}}{N_s}\right) + t_{ft}\left(S_h + S_m\right)\right] \quad (18)$$

where:
T_t is the total turning cycle time (min.);
C_t is the cutting time;
t_a is the approximation and removal tool time;
t_s is the secondary time;
t_p is the setup time;
T is the tool life;
t_{ft} is the tool change time;
Z is the batch size (units);
S_m is the machine cost (US$);
S_h is the labor cost (US$);
V_{si} is the cost of the internal and external tool holder (US$);
K_{pi} is the tool price (US$);
N_{fp} is the average tool life holders;
N_s is the number of the tool's cutting edges.

The decision variables were analyzed in a coded way in order to reduce the variance. Only at the end of the analyses were they converted to their uncoded values. The parameters used in the experiments and their levels are shown in Table 3.

Table 3. Parameters used in the experiments (SEVERINO et al., 2012).

Factors	Levels				
	-1.41	-1	0	1	1.41
Feed (mm rev^{-1})	0.31	0.32	0.35	0.38	0.39
Rotation (rpm)	227	235	255	275	283

The analysis of experimental data shown in Table 2 generated the mathematical modeling presented in Table 4. An excellent fit can be observed, once adjusted R^2 is higher than 90% for all responses.

Table 4. Mathematical models for the objective functions (SEVERINO et al., 2012).

Terms	Life of cutting tool (mm)	Productivity (part hour^{-1})	Cost (US$ part^{-1})
Constant	3,003	1,713	0.03626
Feed	83	90	-0.00293
Rotation	-366	75	0.00054
Feed * Feed	-638	-15	0.00476
Rotation * Rotation	-300	-8	0.00200
Feed* Rotation	-263	-5	0.00236
MSE		12.9	
P-value	0.003	0.000	0.000
Regression (Full quadratic)	0.000	0.000	0.000
Lack of fit	0.926	0.136	0.279
Adjusted R^2 (%)	91.10%	99.90%	94.30%
Normality of residuals	0.600	0.511	0.552

Based on the data presented in Table 4, we start applying the weighting method proposed in this paper. It is important to mention that Tables 2 and 4 are equivalent to Steps 1 and 2, respectively, as described in this study.

To implement the optimization routine described in Step 3, the payoff matrix was estimated initially, obtaining the results reported in Table 5. Based on the payoff matrix, it was possible to iteratively implement Equation (12), choosing w in the range [0;1]. Using this equation, and the parameters from Das and Dennis (1998), 70 points were achieved and the Pareto frontier was built, as shown in Figure 7.

Table 5. Payoff matrix for the objective functions.

Life of cutting tool (mm)	Productivity (part hour^{-1})	Cost (US$ part^{-1})
3,140	1,675	0.03612
1,524	1,850	0.04298
3,067	1,718	0.03558

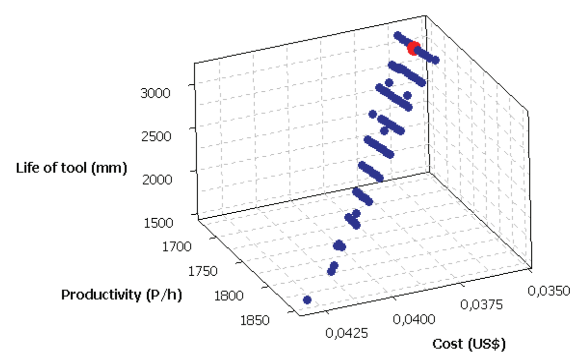


Figure 7. Pareto frontier by NBI method.

Once Step 3 was implemented, an arrangement of mixtures for the weights of each objective function (Step 4) was defined. Subsequently, the solution of the optimization problem of Step 3 was obtained for each experimental condition defined by the arrangement of mixtures (Step 5). Based on these results, we calculated the GPE and the Entropy (Step 6).

From the calculation of the Entropy and GPE, we proceeded to the mathematical modeling of functions (Step 7). Thus, the functions Entropy and GPE are presented by Equations (19) and (20).

$$\begin{aligned} Entropy &= -0,0074w_1 - 0,0074w_2 - 0,0074w_3 + 2,7705w_1w_2 \\ &+ 2,7705w_1w_3 + 2,7705w_2w_3 + 5,4207w_1w_2w_3 + 1,4619w_1w_2(w_1 - w_2)^2 \\ &+ 1,4619w_1w_3(w_1 - w_3)^2 + 1,4619w_2w_3(w_2 - w_3)^2 \end{aligned} \quad (19)$$

$$\begin{aligned} GPE &= 0,1075w_1 + 0,7172w_2 + 0,0920w_3 - 0,0065w_1w_2 + 0,0077w_1w_3 \\ &- 0,0144w_2w_3 - 0,5262w_1w_2w_3 + 0,1134w_1w_2(w_1 - w_2) - 0,0289w_1w_3(w_1 - w_3) \\ &- 0,0452w_2w_3(w_2 - w_3) + 0,7432w_1^2w_2w_3 + 0,9584w_1w_2^2w_3 \\ &- 0,1211w_1w_2(w_1 - w_2)^2 + 0,0367w_1w_3(w_1 - w_3)^2 + 0,0032w_2w_3(w_2 - w_3)^2 \end{aligned} \quad (20)$$

Lastly, Step 8 was executed. By the maximization of ξ, described in Equation (15), the optimal weights w_1, w_2 and w_3, were found. The values are: w_1 (weight of Life of cutting tool) = 0.4976; w_2 (weight of Productivity) = 0.0000; and w_3 (weight of Cost) = 0.5024. The Figure 7 shows the Pareto frontier built using the NBI method, with the optimal highlighted.

These optimal weights were used in a multiobjective optimization of life of the cutting tool, productivity, and cost, as Equation (12), reaching the values of 3,103, 1,697 and 0.03594, respectively. The optimal values of the decision variables are: Feed rate = 0.35 mm rev^{-1} and Rotation = 248 rpm.

In Figure 7, we can see that the distribution of Pareto optimal points on the frontier is evenly distributed. Moreover, we can find in Figure 7 the best-fitted point (the highlighted one in the figure), considering as selection criteria Entropy and GPE. With our proposal, we discovered the optimal point in the frontier that was, at the same time, the more diversified one and the one with the lowest error when comparing the ideal value for each objective function.

Conclusion

This study aimed to propose a method that can identify the optimal weights involved in a multiobjective formulation, in a non-subjective manner. The lack of studies proposed to this end are evidence of this study's relevance. The definition of these weights is also important because this information can be useful for the decision maker in decision-making process.

Thus, this paper has presented a methodology for defining the optimal weights that, by using the design of experiments (DOE), has generated optimum values for the decision variables that can be implemented in the vertical turning process analyzed herein. Despite it being a method with a relatively large number of 'steps', it presents itself as easy to implement, without generating large computational demand, since the tools are available in popular software such as the Solver function of Excel® and Minitab®.

The Entropy and the Global Percentage Error (GPE) function, used as a criterion for evaluating Pareto-optimal solutions, were identified as suitable indicators, enabling their modeling via a polynomial of mixtures that delimited a region of maximum diversification and minimum error for the weight combination analyzed.

Another finding in this study was the possibility of constructing, in an easy way, an evenly distributed Pareto frontier for more than two objectives. With the present proposal, the Entropy and the GPE can be calculated for any number of objective functions. Besides, the Pareto frontier and the optimal weights can be reached using the NBI method as described. This is an advantage, mainly when the computational economy is considered.

In the analyzed process, the optimal parameters 'Feed rate = 0.35 mm rev^{-1}' and 'Rotation = 248 rpm' give us a maximum tool life and minimum cost region. It is important to see that increasing tool life is a way to reduce cost. This is true, mainly in this process, because of the features of the material.

Thus, the main contributions of this study are the proposal of a structured method, differentiated in relation to the techniques found in the literature, of identifying optimal weights for multiobjective problems and the possibility of viewing the optimal result along the Pareto frontier of the problem. This viewing possibility is very relevant information for the more efficient management of processes. Moreover, it can be stated that the proposed method promotes maximum achievement among multiple objectives, i.e., between a set of Pareto-optimal solutions, being able to identify the best optimal, based on the aforementioned selection criteria.

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